OCDSB NIGHT SCHOOL
CONTINUING EDUCATION
PRACTICE EXAM

Mathematics Department

Including this cover page, this exam has __ pages.

Date: Tuesday May 23, 2017

Time: 6:00 PM to 9:00 PM

Overall Expectations: Advanced Functions (MHF4U)
By the end of this course, students will:

A. EXPONENTIAL AND LOGARITHMIC FUNCTIONS
A1. Demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify numeric expressions;

A2. Identify and describe key features of the graphs of logarithmic functions, make connections among the numeric, graphical , and algebraic representations of logarithmic functions, and solve related problems graphically;

A3. Solve exponential and simple logarithmic equations in one variable algebraically, including those in problems arising from real-world applications.

B. TRIGONOMETRIC FUNCTIONS
B1. Demonstrate an understanding of the meaning and application of radian measure;

B2. Make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric function and their reciprocals, and use these connections to solve problems;

B3. Solve problems involving trigonometric equations and prove trigonometric identities.

C. POLYNOMIAL AND RATIONAL FUNCTIONS
C1. Identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;

C2. Identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;

C3. Solve problems involving polynomial and simple rational equations graphically and algebraically;

C4. Demonstrate an understanding of solving polynomial and simple rational inequalities.

D. CHARACTERISTICS OF FUNCTIONS
D1. Demonstrate an understanding of average and instantaneous rate of change, and determine numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;

D2. Determine functions that result from the addition, subtraction, multiplication, and division of two functions, and from the composition of two functions, describe some properties of the resulting two functions, and solve related problems;

D3. Compare the characteristics of functions, and solve problems by modeling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

Evaluation

This examination consists of three (4) sections – one section for each of the strands A: *Exponential and Logarithmic Functions*, B *Trigonometric Functions*, C *Polynomial and Rational Functions* and D: *Characteristics of Functions*.

Students will be evaluated in each strand on the extent to which they meet the criteria set out, with respect to the strand, for the following: Knowledge and Understanding, Thinking and Inquiry, Communication, and Application criteria listed in the rubric below.

Students who fail to meet Level 1 in a strand will be assessed a Level R for that strand.

Categories	50-59% (Level 1)	60-69% (Level 2)	70-79% (Level 3)	80-100% (Level 4)
Knowledge and Understanding: Content knowledge and comprehension of meaning and significance				
Knowledge of Content (facts, terms, proper procedural steps)	demonstrates **limited knowledge** of content	demonstrates **some knowledge** of content	demonstrates **considerable knowledge** of content	demonstrates **thorough knowledge** of content
Understanding of Mathematical Concepts (proper procedures used)	demonstrates **limited understanding** of concepts	demonstrates **some understanding** of concepts	demonstrates **considerable understanding** of concepts	demonstrates **thorough understanding** of concepts
Thinking: use of critical and creative thinking skills and/or processes				
Use of Processing Skills (reasonableness, justifying, proving, reflecting)	uses processing skills with **limited effectiveness**	uses processing skills with **some effectiveness**	uses processing skills with **considerable effectiveness**	uses processing skills with a **high degree of effectiveness**
Communication: The conveying of meaning through various forms				
Use of Conventions, Vocabulary, and Terminology in Visual, and Written Forms (units, symbols, terms)	uses conventions, vocabulary, and terminology of the discipline with **limited effectiveness**	uses conventions, vocabulary, and terminology of the discipline with **some effectiveness**	uses conventions, vocabulary, and terminology of the discipline with **considerable effectiveness**	uses conventions, vocabulary, and terminology of the discipline with a **high degree of effectiveness**
Application: Make connections between and within various contexts using knowledge and skills				
Application of Knowledge and Skills in Familiar Contexts	applies knowledge and skills in familiar contexts with **limited effectiveness**	applies knowledge and skills in familiar contexts with **some effectiveness**	applies knowledge and skills in familiar contexts with **considerable effectiveness**	applies knowledge and skills in familiar contexts with a high **degree of effectiveness**

Each section consists of two parts:

- **Short Answer**
- **Extended Response**

<u>Short Answer</u>:
The final answer will be valued; however, considerations may be given to work shown.

<u>Extended Response</u>:
Students are expected and required to organize and express complete responses to each of the problems such that they demonstrate the full range of their understanding of the relevant mathematical concept(s). Students will provide logical justification for their conclusions and ideas and use representations (algebraic, numerical, and graphical) when they communicate their ideas.

Section A: Logarithmic and Exponential Functions

Short Answer:

1. **State** the transformations applied to $f(x) = \log 2x + 5$ to obtain $g(x) = 10\log(2x - 3) + 4$.
2. Explain the relationship between a log function and an exponential function.
3. Given $\log_2 a + \log_2 b = 4$, what are the possible values of a and b (a, b $\in \aleph$)?
4. **Evaluate** $\log_9 1200$ to 3 decimal places

Extended Response:

5. **Solve:** $8^{3x-7} = 11^{-x-1}$ to 3 decimal places
6. **Solve:** $8^{3x-7} = 64^{-x-1}$ to 3 decimal places
7. A bacteria culture starts with 10 000 bacteria. After 40 minutes the count is 30 000. What is the doubling period?
8. **Solve and check** $\log(3x+1) + \log(x-1) = \log(6x-1)$
9. **Graph** $f(x) = \log 2x + 5$ and it's inverse
10. Evaluate the following logarithms to 3 decimal places. Explain the pattern in the results.
    a) log 5
    b) log 25
    c) log 125
    d) log 625
    e) log 3125

Section B: Trigonometric Functions

Short Answer:

1. State the **exact** value of $\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{2}\right)$
2. Describe the key features of $f(x) = \tan x$
3. Solve $\csc(x) = -\frac{2}{\sqrt{3}}$, $0 \le x \le 2\pi$.
4. Write an equation to represent a periodic function with amplitude 8, period π, and with one of its maxima at (0,-5).

Extended Response:

5. Solve: $\cos(3x + \pi) = -\frac{1}{2}$ in the interval $-2\pi \le x \le 2\pi$.
6. α and β are acute angles in quadrant I, with $\sin\alpha = \frac{2}{7}$ and $\cos\beta = \frac{3}{5}$. **Without** finding the size of the angles or using decimal values, determine the value of $\sin(\alpha + \beta)$ and $\tan(\alpha + \beta)$
7. A pedal on a bicycle has an arm length of 20 cm and rotates about an axle 32 cm above the ground. If the pedal starts at its lowest point and rotates at 20 revolutions every minute, find a sinusoidal function that will model the height *h*, in centimetres, of the pedal after *t* seconds. At what time during the first 5 seconds will the pedal be 40 cm above the ground?

Section C: Polynomial and Rational Functions

Short Answer:

1. Determine the equation that contains the following information:
   - degree 4, roots –2 and -1, 1, and 3, with a y-intercept of -3 (**Write final answer in factored form – do not expand**)
2. Write the equations of **all** asymptotes and holes for the function**:** $\dfrac{x^2-5x+6}{x^2-9}$
3. Write a rational equation that cannot have 6 or -8 as solutions

Extended Response:

4. Fully factor the functions:
   a. $f(x)=x^4+x^3-7x^2-1x+6$
   b. $g(x)=x^4-27x^2-14x+120$
5. Solve the rational inequality: $\dfrac{x+8}{x-5}\le 2x+1$
6. Solve the following polynomial: $2x^4+x^3-14x^2+5x+6=0$
7. Graph the following polynomial: $f(x)=2x^4+x^3-14x^2+5x+6$
8. An open box is made from a rectangular piece of cardboard with dimensions 18 cm by 25 cm, by cutting congruent squares from each corner and folding up the sides. Determine all possible dimensions of the squares to be cut to create a volume less than 189 cubic cm.

Section D: Characteristics of Functions

Short Answer:

1. A golfer hits a ball up in the air. The height of the golf ball in metres above the ground is given by $h(x)=-4.9x^2+4x+0.1$, where *t* is the number of seconds after the ball is hit. What is the average rate of change in the height of the ball over the interval $1.5\le x\le 2$? Does this make sense?
2. Given $f(x)=4x^2-3x$ and $g(x)=\dfrac{4}{2x-7}$, determine the following. State the **domain and range** then **graph** each combined function.

a) $(f+g)(x)$	b) $g(f(x))$
c) $(f-g)(x)$	d) $f(g(x))$
e) $f(x)g(x)$	f) $f(x)\div g(x)$

Extended Response:

3. A particle moves along the y axis with the relationship $s(t)=-0.5t^3+9t^2 48t+64$ where *s(t)* represents the displacement and *t* represents the time.
   a. What is the instantaneous rate of change (velocity) at t = 2
   b. Is there a maximum, minimum, or neither at t = 2? Give reasoning.
4. Given that f(x) = -9x+3 and g(x) = $\sqrt{3x+7}$, evaluate

a) $f\circ g(20)$	b) $g\circ f(4)$
c) $f(3)+g(7)$	d) $f(3)\bullet g(7)$
e) $f(3)+g^{-1}(7)$	f) $f^{-1}(f(3))$

MHF4U Exam Formula Sheet

Trig Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double Angle Formulae

$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Addition & Subtraction Formulae

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Richter Scale

$$\mathrm{M} = \log\left(\frac{\mathrm{I}}{\mathrm{I}_0}\right)$$

dB

$$L = 10\log\left(\frac{I}{I_o}\right)$$

PH

$$\mathrm{pH} = -\log[\mathrm{H}^+]$$

Half Life

$M = M_o(1/2)^{t/h}$

$I_o = 10^{-12}$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$